

NAG C Library Function Document

nag_zpbstf (f08utc)

1 Purpose

nag_zpbstf (f08utc) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix.

2 Specification

```
void nag_zpbstf (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer kb,
                 Complex bb[], Integer pdbb, NagError *fail)
```

3 Description

nag_zpbstf (f08utc) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix B . It is designed to be used in conjunction with nag_zhbgst (f08usc).

The factorization has the form $B = S^H S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n + k)/2$ rows, and transposed hence, lower triangular in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & & & s_{66} & & & \\ & & & s_{64} & s_{65} & s_{66} & & & \\ & & & & s_{75} & s_{76} & s_{77} & & \\ & & & & & s_{86} & s_{87} & s_{88} & \\ & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

4 References

None.

5 Parameters

- 1: **order** – Nag_OrderType *Input*
On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.
Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.
- 2: **uplo** – Nag_UploType *Input*
On entry: indicates whether the upper or lower triangular part of B is stored as follows:
 - if **uplo** = **Nag_Upper**, the upper triangular part of B is stored;
 - if **uplo** = **Nag_Lower**, the lower triangular part of B is stored.*Constraint:* **uplo** = **Nag_Upper** or **Nag_Lower**.

- 3: **n** – Integer *Input*
On entry: n , the order of the matrix B .
Constraint: $n \geq 0$.
- 4: **kb** – Integer *Input*
On entry: k , the number of super-diagonals of the matrix B if **uplo** = **Nag_Upper**, or the number of sub-diagonals if **uplo** = **Nag_Lower**.
Constraint: $kb \geq 0$.
- 5: **bb**[*dim*] – Complex *Input/Output*
Note: the dimension, *dim*, of the array **bb** must be at least $\max(1, \mathbf{pdbb} \times \mathbf{n})$.
On entry: the n by n Hermitian band matrix B . This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements b_{ij} depends on the **order** and **uplo** parameters as follows:
- if **order** = **Nag_ColMajor** and **uplo** = **Nag_Upper**,
 b_{ij} is stored in **bb**[$k + i - j + (j - 1) \times \mathbf{pdbb}$], for $i = 1, \dots, n$ and
 $j = i, \dots, \min(n, i + k)$;
 - if **order** = **Nag_ColMajor** and **uplo** = **Nag_Lower**,
 b_{ij} is stored in **bb**[$i - j + (j - 1) \times \mathbf{pdbb}$], for $i = 1, \dots, n$ and
 $j = \max(1, i - k), \dots, i$;
 - if **order** = **Nag_RowMajor** and **uplo** = **Nag_Upper**,
 b_{ij} is stored in **bb**[$j - i + (i - 1) \times \mathbf{pdbb}$], for $i = 1, \dots, n$ and
 $j = i, \dots, \min(n, i + k)$;
 - if **order** = **Nag_RowMajor** and **uplo** = **Nag_Lower**,
 b_{ij} is stored in **bb**[$k + j - i + (i - 1) \times \mathbf{pdbb}$], for $i = 1, \dots, n$ and
 $j = \max(1, i - k), \dots, i$.
- On exit:* B is overwritten by the elements of its split Cholesky factor S .
- 6: **pdbb** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix B in the array **bb**.
Constraint: $\mathbf{pdbb} \geq \mathbf{kb} + 1$.
- 7: **fail** – NagError * *Output*
The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: $\mathbf{n} \geq 0$.

On entry, **kb** = $\langle value \rangle$.

Constraint: $\mathbf{kb} \geq 0$.

On entry, **pdbb** = $\langle value \rangle$.

Constraint: $\mathbf{pdbb} > 0$.

NE_INT_2

On entry, **pdbb** = $\langle value \rangle$, **kb** = $\langle value \rangle$.

Constraint: $\mathbf{pdbb} \geq \mathbf{kb} + 1$.

NE_CONVERGENCE

The factorization could not be completed, because updated element $b(\langle value \rangle, \langle value \rangle)$ would be the square root of a negative number. Hence B is not positive definite. This may indicate an error in forming the matrix B .

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix $B + E$, where

$$|E| \leq c(k+1)\varepsilon|S^H||S|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ε is the *machine precision*. It follows that $|e_{ij}| \leq c(k+1)\varepsilon\sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $4n(k+1)^2$, assuming $n \gg k$.

A call to this function may be followed by a call to `nag_zhbgst (f08usc)` to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive-definite.

The real analogue of this function is `nag_dpbstf (f08ufc)`.

9 Example

See Section 9 of the document for `nag_zhbgst (f08usc)`.